

1. NO CALCULATORS ALLOWED
2. UNLESS OTHERWISE INSTRUCTED, SIMPLIFY ALL ANSWERS COMPLETELY
3. SHOW PROPER & CONCISE PRECALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

[FILL IN THE BLANKS]

SCORE: \_\_\_\_ / 7 PTS

- [a] You start at the origin in 3D, and move 11 units right, 9 units down, and 12 units backward. You are now at the point with

co-ordinates  $(-12, 11, -9)$ , you are in octant 6, and you are 12 units away from the  $yz$ -plane.

- [b] If  $\vec{a} \cdot \vec{b} = 9$ , then the angle between  $\vec{a}$  and  $\vec{b}$  is ACUTE.

- [c] The equation of the  $xz$ -trace of the sphere  $(x+2)^2 + (y-3)^2 + (z-1)^2 = 19$  is  $(x+2)^2 + (z-1)^2 = 10$ .

Three forces act on an object.

SCORE: \_\_\_\_ / 8 PTS

Force 1 has magnitude 8 newtons and direction angle  $60^\circ$ .

Force 2 has magnitude 12 newtons and direction angle  $150^\circ$ .

Force 3 has magnitude 5 newtons and direction angle  $270^\circ$ .

- [a] Find the resultant of the three forces. Write your answer as a linear combination of  $\vec{i}$  and  $\vec{j}$ .

$$\begin{aligned}
 & 8 \langle \cos 60^\circ, \sin 60^\circ \rangle \\
 & + 12 \langle \cos 150^\circ, \sin 150^\circ \rangle \\
 & + 5 \langle \cos 270^\circ, \sin 270^\circ \rangle \\
 & = \langle 4, 4\sqrt{3} \rangle + \langle -6\sqrt{3}, 6 \rangle + \langle 0, -5 \rangle \\
 & = \langle 4 - 6\sqrt{3}, 1 + 4\sqrt{3} \rangle \\
 & = \left( \frac{1}{2} \right) (4 - 6\sqrt{3}) \vec{i} + (1 + 4\sqrt{3}) \vec{j}
 \end{aligned}$$

① FOR THIS FORMAT (USING  $\vec{i}, \vec{j}$ )

- [b] The resultant of the three forces acted on the object as it moved from  $(-3, 2)$  to  $(-1, -4)$ , where all coordinates are measured in meters. Find the work done, and give appropriate units for your answer.

$$\vec{d} = \langle -1 - (-3), -4 - 2 \rangle = \langle 2, -6 \rangle$$

$$\langle 4 - 6\sqrt{3}, 1 + 4\sqrt{3} \rangle \cdot \langle 2, -6 \rangle = 8 - 12\sqrt{3} - 6 - 24\sqrt{3}$$

①

$$= 2 - 36\sqrt{3} \text{ Nm or J}$$

①

① FOR EITHER ANSWER

Write  $\vec{g} = \langle 11, 7 \rangle$  as the sum of 2 vectors, one perpendicular to  $\vec{d} = \langle -4, 2 \rangle$  and one parallel to  $\vec{d}$ .

SCORE: \_\_\_\_ / 5 PTS

$$\text{PROJ}_{\vec{d}} \vec{g} = \frac{\langle 11, 7 \rangle \cdot \langle -4, 2 \rangle}{\langle -4, 2 \rangle \cdot \langle -4, 2 \rangle} \langle -4, 2 \rangle = \frac{-30}{20} \langle -4, 2 \rangle = -\frac{3}{2} \langle -4, 2 \rangle = \langle 6, -3 \rangle$$

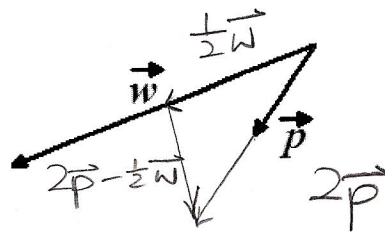
$$\langle 11, 7 \rangle - \langle 6, -3 \rangle = \langle 5, 10 \rangle$$

$$\langle 11, 7 \rangle = \langle 6, -3 \rangle + \langle 5, 10 \rangle$$

FOR THIS FORMAT / EQUATION

For the vectors shown below, sketch the vector  $2\vec{p} - \frac{1}{2}\vec{w}$ .

SCORE: \_\_\_\_ / 2 PTS



Let  $\vec{d} = 6\vec{j} - 2\sqrt{3}\vec{i}$ .

SCORE: \_\_\_\_ / 8 PTS

- [a] Find a vector  $\vec{s}$  in the opposite direction as  $\vec{d}$ , such that  $\|\vec{s}\| = 3$ . Write your answer in component form.

$$\begin{aligned} -\frac{3}{\|\vec{d}\|} \vec{d} &= -\frac{3}{\sqrt{12+36}} \langle -2\sqrt{3}, 6 \rangle \\ &= -\frac{3}{\sqrt{48}} \langle -2\sqrt{3}, 6 \rangle \\ &= -\frac{3}{4\sqrt{3}} \langle -2\sqrt{3}, 6 \rangle = \left\langle \frac{3}{2}, -\frac{3\sqrt{3}}{2} \right\rangle \end{aligned}$$

- [b] If  $\vec{p}$  is a vector with magnitude 6 which makes an angle of  $135^\circ$  with  $\vec{d}$ , find  $\vec{p} \cdot \vec{d}$ .

$$\begin{aligned} \|\vec{p}\| \|\vec{d}\| \cos \theta &= 6(4\sqrt{3}) \cos 135^\circ \\ &= 6(4\sqrt{3}) \left(-\frac{\sqrt{2}}{2}\right) \\ &= -12\sqrt{6} \end{aligned}$$

- [c] Find the direction angle of  $\vec{d}$ .

$$\begin{aligned} \theta_d &= \pi + \tan^{-1} \frac{6}{-2\sqrt{3}} \\ &= \pi + \tan^{-1}(-\sqrt{3}) = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \end{aligned}$$